Idealized Numerical Modeling of Internal Wave Propagation Through Density Staircases

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1. Summary

- **How do vertical density structures affect internal wave propagation?**
- Use direct simulations of internal waves through 1 or 2 mixed layers
- Reproduce transmission and reflection observed in laboratory experiments
- Use complex demodulation to separate up and downward propagation

2. Background

- Arctic Ocean contains warm Atlantic water below pycnocline with enough heat to melt all Arctic sea ice[5]

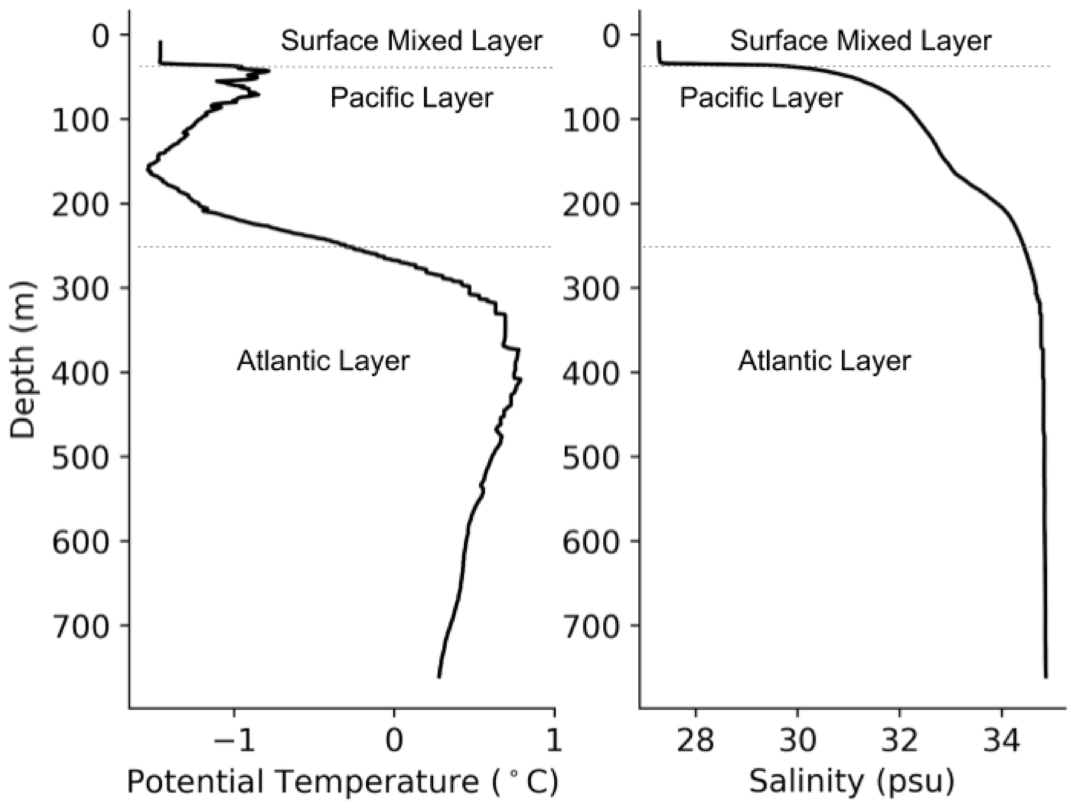

Fig. 1: Typical profile measurements of the Arctic Ocean's Canadian Basin. Adapted[4]

- Winds on surface create internal waves which can cause vertical mixing

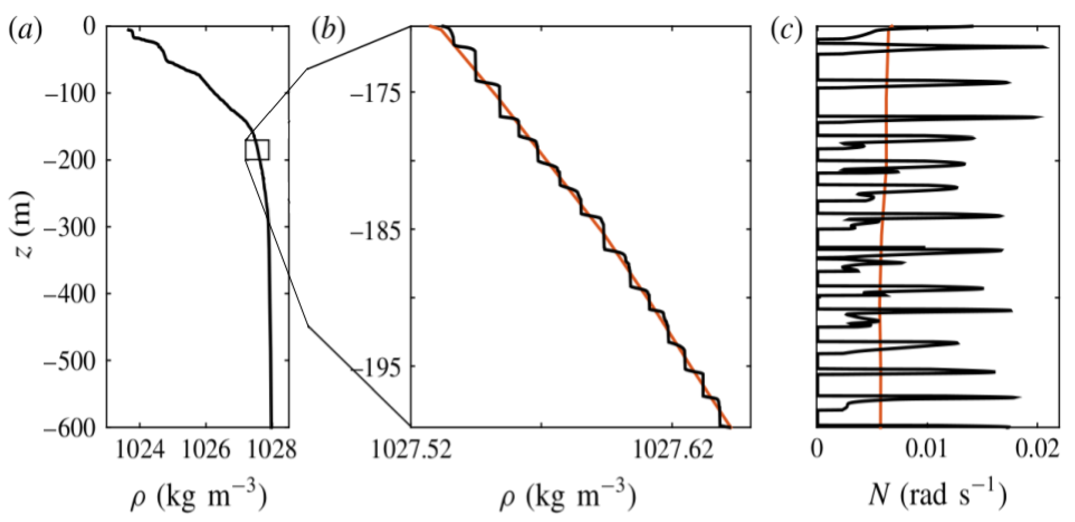

Fig. 2: Typical density and stratification profiles in Canadian Basin showing double diffusive staircases in Atlantic Layer. Reproduced[2]

3. Results

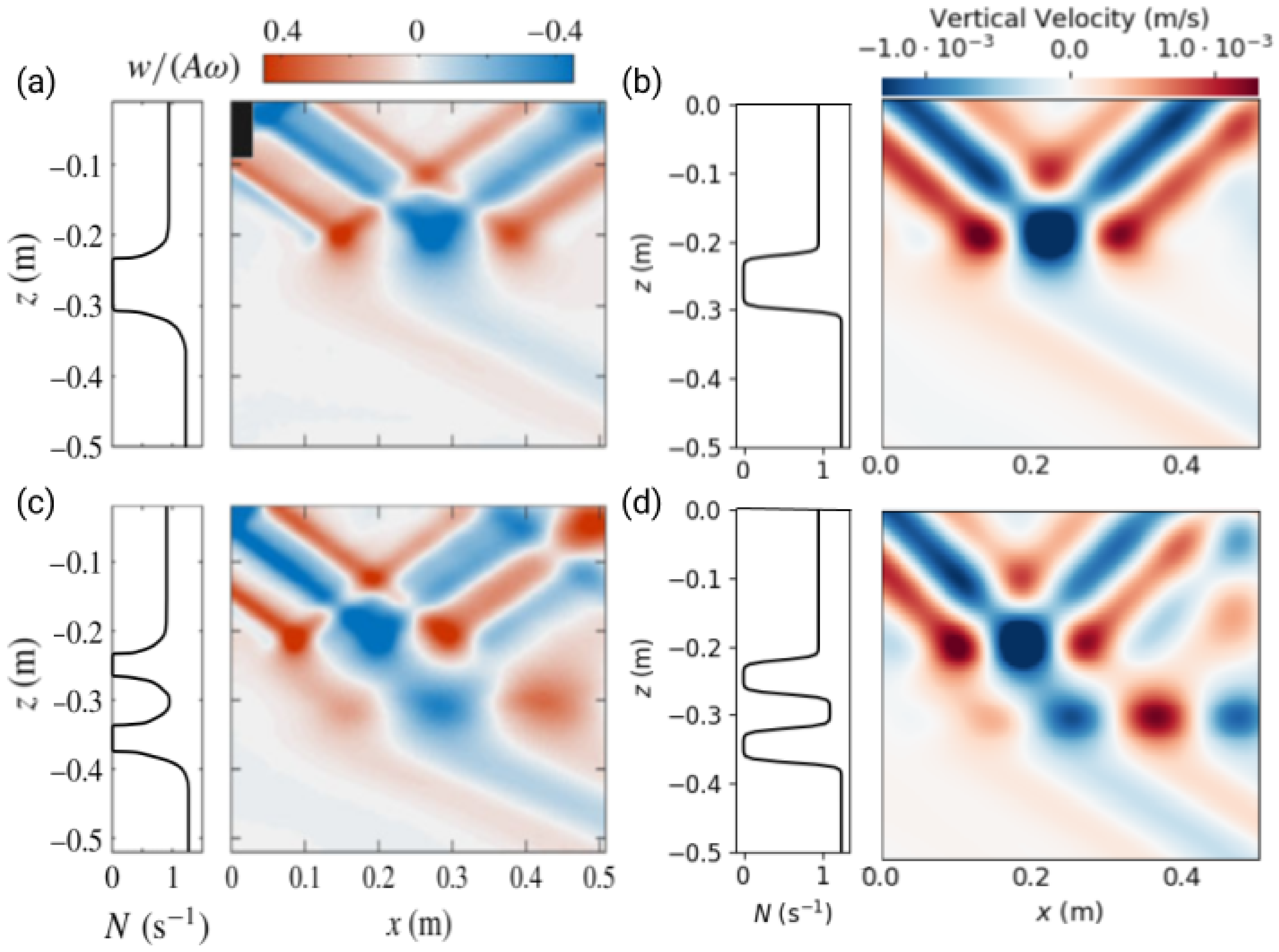

Fig. 3: The experimental results from ENDLab for (a) single and (c) double mixed layer. Adapted[2]. A direct numerical simulation the (b) single and (d) double mixed layer conditions of the wave tank.

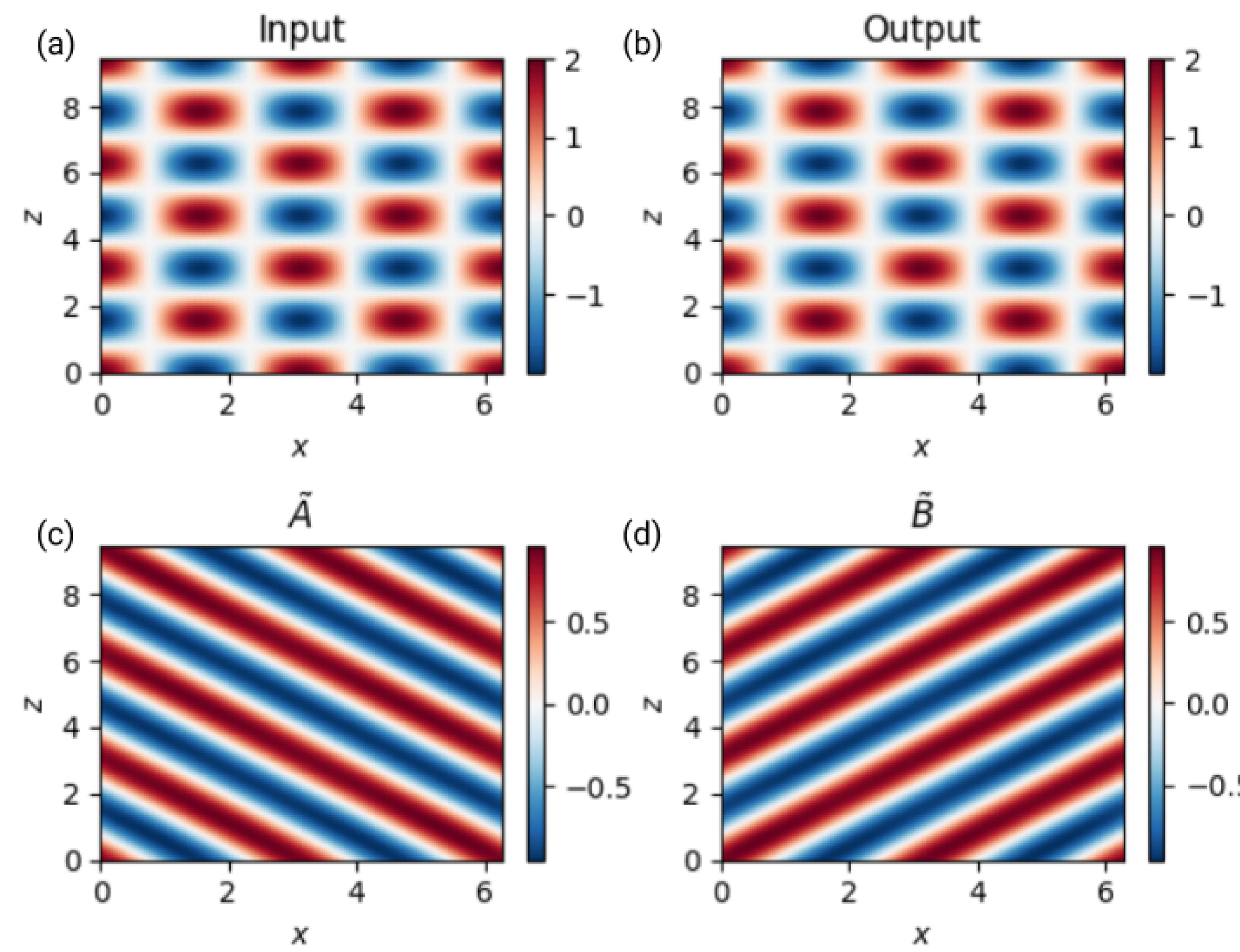

Fig. 4: CD of (a) $U = A \exp i(\omega t - k_x x - k_z z) + B \exp i(\omega t - k_x x + k_z z)$, an ideal input wave separating (c) up and (d) downward propagating components which sum to (b) the output wave, identical to the input.

4. Methods

Direct Numerical Simulation

Use Boussinesq equations of motion in Dedalus[1] and force plane waves:

$$\vec{v}(x,z,t) = \vec{v}'(x,z,t) \exp\ i(\omega t - k_x x - k_z z)$$

Dispersion relation predicts evanescent waves when $\omega > N_0$:

$$k_z^2 = k_x^2 \left(\frac{N_0^2}{\omega^2} - 1 \right)$$

Complex Demodulation[3] (CD)

- Assume plane waves
- Fourier transform in time, filter in frequency space for ω
- Fourier transform in space, filter by sign of wavenumbers k_x and k_z
- Inverse Fourier transform to separate up and downward propagation

5. Future Work

- Apply CD to wave tank simulation
- Compare calculated transmission and reflection to theoretical prediction[2]
- Add more mixed layers to profile
- Use ω, k, and N typical of Arctic Ocean

Acknowledgements

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References

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